PROMETHEUS

PRivacy preserving pOst-quantuM systEms from advanced crypTograpHic mEchanisms Using latticeS

Tools for Parameter Selection

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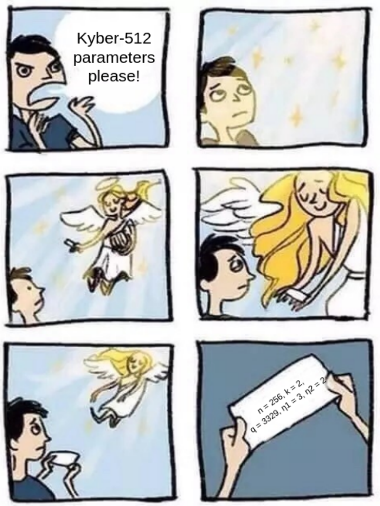
Parameters are ‘god given’



Parameters are ~~‘god given’~~ a human work

This presentation concerns the tools that have been {created, expanded, maintained} under the PROMETHEUS project – what they do, how they work, and how they have been used.

Specifically:

- the lattice-estimator [APS15] (`github:malb/lattice-estimator`),
- the leaky-LWE-estimator [DDGR20] (`github:lducas/leaky-LWE-Estimator`),
- NTRUFatigue-estimator [DvW21] (`github:WvanWoerden/NTRUFatigue`).

The high level *what*

Broadly speaking, any (lattice) estimator takes as input a subset of

$$\{parameters\} \times \{attacks\} \times \{attack\ models\},$$

and outputs estimates for the costs of the attacks. For example

$$\{\text{Kyber512}\} \times \{\text{primal uSVP}\} \times \{\text{cost: ADPS16, shape: GSA}\}.$$

The high level *what*

Different estimators aim for different levels of generality:

- lattice-estimator $\leftrightarrow$ high generality (many parameters, attacks, models),
- leaky-LWE-estimator $\leftrightarrow$ high specificity (some parameters, one attack in detail),[1]
- NTRUFatigue-estimator $\leftrightarrow$ high specificity (some parameters, one attack in detail).

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Ultimately, the aim is to integrate all impactful attacks into the lattice-estimator.

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The lattice-estimator

An implementation that came out of a paper [APS15] that systematised and improved the analysis of a large range of attacks against LWE.

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The aim is to capture as many attacks and improvements as possible and automate their cost estimation against a wide range of popular parameters, e.g. dimension, modulus, secret and error distributions etc...

How to use: the lattice-estimator

Create parameter object.

```
sage: from estimator import *
....: params = LWE.Parameters(n=200, q=7981, Xs=ND.SparseTernary(384, 16), Xe=ND.CenteredBinomial(4))
```

Estimate suite of attacks given:

- a model for lattice reduction *cost*,
- and a model for lattice reduction *shape*.

How to use: the lattice-estimator

Then

```
sage: r = LWE.estimate(params, red_cost_model=RC.MATZOV, red_shape_model=Simulator.GSA)
```

gives

```
arora-gb             :: rop: ≈2^102.2, dreg: 9, mem: ≈2^101.2, t: 4, m: ≈2^51.6, tag: arora-gb, ↻: 2, ζ: 1, |S|: 1, prop: 1
bkw                  :: rop: ≈2^52.0, m: ≈2^41.6, mem: ≈2^42.5, b: 3, t1: 0, t2: 12, ℓ: 2, #cod: 155, #top: 1, #test: 44, tag: coded-bkw
usvp                 :: rop: ≈2^50.4, red: ≈2^50.4, δ: 1.010720, β: 72, d: 331, tag: usvp
bdd                  :: rop: ≈2^47.9, red: ≈2^47.1, svp: ≈2^46.5, β: 60, η: 87, d: 345, tag: bdd
bdd_hybrid           :: rop: ≈2^46.3, red: ≈2^46.2, svp: ≈2^41.7, β: 40, η: 22, ζ: 51, |S|: ≈2^12.3, d: 324, prob: 0.141, ↻: 31, tag: hybrid
bdd_mitm_hybrid      :: rop: ≈2^48.2, red: ≈2^47.7, svp: ≈2^46.4, β: 40, η: 2, ζ: 114, |S|: ≈2^47.1, d: 261, prob: 0.041, ↻: 111, tag: hybrid
dual                 :: rop: ≈2^51.9, mem: ≈2^18.8, m: 174, β: 73, d: 374, ↻: 1, tag: dual
dual_hybrid          :: rop: ≈2^46.6, mem: ≈2^35.5, m: 145, β: 40, d: 287, ↻: 21, ζ: 58, h1: 4, tag: dual_hybrid
```

How to use: the lattice-estimator

One can also specific a particular attack and its options

```
sage: LWE.primal_hybrid(params, mitm=False, babai=False, red_shape_model=Simulator.CN11)
```

to obtain

```
rop: ≈2^46.6, red: ≈2^45.6, svp: ≈2^45.6, β: 42, η: 22, ζ: 52, |S|: ≈2^17.5, d: 330, prob: 0.308, ↻: 13, tag: hybrid
```

Most importantly, read the docs!
https://lattice-estimator.readthedocs.io/en/latest/.

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Most importantly, *write* the docs!
`https://lattice-estimator.readthedocs.io/en/latest/`.

How it works: the lattice-estimator

It has a highly modularised codebase

- top level Estimate class $\leftrightarrow$ lwe.py,
- initiating and transforming LWE parameters $\leftrightarrow$ lwe_parameters.py,
- each attack has its own file, with a class per variant $\leftrightarrow$ e.g. lwe_primal.py with class PrimalHybrid,
- reduction cost models $\leftrightarrow$ reduction.py,
- reduction shape models $\leftrightarrow$ simulator.py,
- error and secret (‘noise’) distributions $\leftrightarrow$ nd.py,
- several more, probability amplification, distinguishing, mitm...

Ultimately, the estimator uses a local minimum finder to minimise the cost of a given attack in a given model against given parameters.

Some useful notes...

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We *do* have a good understanding of which β succeed for different attacks, what is challenging is converting this into a more expressive cost.

The leaky-LWE-estimator

So called because its main function is the integration of leaks (i.e. side channel information) into the primal uSVP attack.

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There are excellent tutorials in [DDGR20, App. A] for the side channel aspects, I will focus on the probabilistic uSVP estimator.

How to use: the leaky-lwe-estimator

Set up parameters

```
sage: load("framework/instance_gen.sage")
sage: n, q, m = 640, 2**15, 640 + 16 # FRODO-640
sage: frodo_distribution = [9288, 8720, 7216, 5264, 3384, 1918, 958, 422, 164, 56, 17, 4, 1]
sage: D_s = get_distribution_from_table(frodo_distribution, 2 ** 16)
sage: D_e = D_s
```

Initialise the instance as inst

```
sage: _, _, inst = initialize_from_LWE_instance(DBDD_predict_diag, n, q, m, D_e, D_s)
```

How to use: the leaky-lwe-estimator

Integrate any short vector hints (we know 'q' vectors will be in our lattice)

```
sage: inst.integrate_q_vectors(q, report_every=20)
          Integrating q-vectors
 [...20]   integrate short vector hint             Unworthy hint, Rejected.
```

Perform estimation *without* probabilism or simulation (e.g. Simulator.GSA of lattice-estimator)

```
sage: inst.estimate_attack()
          Attack Estimation
 dim=1297          δ=1.003474         β=485.84

(485.8376072943104, 1.00347440808773)
```

How to use: the leaky-lwe-estimator

Perform estimation *with* probabilism and simulation

```
sage: inst.estimate_attack(probabilistic=True, ignore_lift_proba=True, lift_union_bound=True)
```

Accumulates probabilities over progressive BKZ tours, and gives

```
β= 495,   pr=1.4798e-01,        cum-pr=3.5544e-01        rem-pr=6.4456e-01
β= 496,   pr=2.0675e-01,        cum-pr=4.8870e-01        rem-pr=5.1130e-01
β= 497,   pr=2.7776e-01,        cum-pr=6.3072e-01        rem-pr=3.6928e-01
β= 498,   pr=3.5929e-01,        cum-pr=7.6340e-01        rem-pr=2.3660e-01
β= 499,   pr=4.4815e-01,        cum-pr=8.6943e-01        rem-pr=1.3057e-01
β= 500,   pr=5.4007e-01,        cum-pr=9.3995e-01        rem-pr=6.0054e-02
β= 501,   pr=6.3022e-01,        cum-pr=9.7779e-01        rem-pr=2.2207e-02
β= 502,   pr=7.1402e-01,        cum-pr=9.9365e-01        rem-pr=6.3506e-03
β= 503,   pr=7.8780e-01,        cum-pr=9.9865e-01        rem-pr=1.3476e-03
β= 504,   pr=8.4928e-01,        cum-pr=9.9980e-01        rem-pr=2.0311e-04
       Attack Estimation
 dim=1297                       β=496.36
```

The NTRUFatigue-estimator

A line of works [ABD16, CJL16, KF17] showed important security implications of setting the modulus q too large in structured lattice schemes. Initial results were asymptotic and experiments focussed on low power lattice reduction and *large* q.

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This work makes huge strides towards concretising the size of q required for these attacks to function, and understanding precisely how lattice reduction functions on such instances.

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This work makes huge strides towards concretising the size of q required for these attacks to function, and understanding precisely how lattice reduction functions on such instances.

As part of the software contribution the authors give a probabilistic estimator that determines an average blocksize β where the first of two potential cryptanalytic events (DSD and SKR) will occur.

How to use: the NTRUFatigue-estimator

Load the estimator and set up the instance

```
sage: load("estimator.sage")
sage: q, n, var, ntru, tours = 2**16, 512, 2./3., "matrix", 8
```

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```

Run the estimator

```
sage: beta, SKR, DSD, position = combined_attack_prob(q, n, sk_variance=var,
 ....:                                                ntru=ntru,
 ....:                                                fixed_tours=tours)
```

How to use: the NTRUFatigue-estimator

Inspect the results

```
sage: beta
194.5987821425533
sage: SKR
-2.049973797868008e-11
sage: DSD
0.9997352694931326
```

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194.59877821425533
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0.9997352694931326
```

Note that if we use a smaller $q = 12289$, we receive

```
sage: beta
285.85405500098994
sage: SKR
0.9998508564732661
sage: DSD
0.0
```

Use cases

A non exhaustive list of how these estimators have been used:

- NIST candidates: KYBER, SABER, FRODO, DILITHIUM,
- PROMETHEUS research: to estimate the security of signatures based on new Gaussian samplers over modules [BEP+21],
- non PROMETHEUS research: to estimate LSH based improvements to MitM attacks on ternary LWE [KM21], in FHE standardisation [ACC+18].

Conclusion

When are these tools for you? Whenever any subset of the following apply:

- you have designed a scheme based on LWE or NTRU,
- you want to understand how lattice attacks behave against concrete parameter choices for it,
- you want to know which attacks to consider (non automatically) in more detail,
- you want to understand the (lattice reduction based) implications of certain kinds of side channels,
- you want to rule out attacks against too large moduli.

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